

CONDITIONAL PROBABILITY

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CONDITIONAL PROBABILITY

Consider the **resting heart rate** in the adult U.S. population

- What is the probability that a randomly selected adult has a resting heart rate below 60 beats per minute ?

Now, suppose **you learn** that the individual is a trained endurance athlete, such as running marathons.

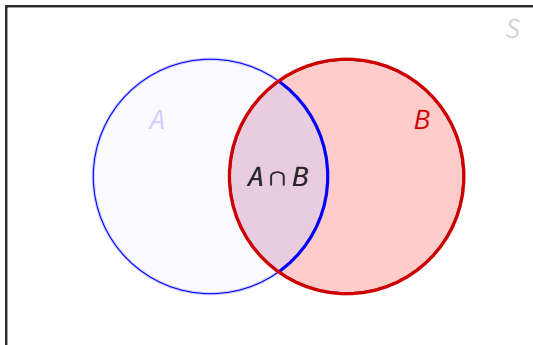
- Does this **new information** change the probability that their resting heart rate is below 60 beats per minute ?

Conditional probability involves calculating the likelihood of an event occurring **given** that partial information is available about the occurrence of another related event.

The conditional probability of outcome A given an even or condition B is

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

whenever $P(B) > 0$.



After conditioning on B , only the portion of A that lies inside B , namely $A \cap B$, matters

GUIDED PRACTICE

Suppose that 10% of adults in the U.S. have a resting heart rate below 60 beats per minute, 20% are trained endurance athletes, and 8% are both trained endurance athletes and have a resting heart rate below 60 beats per minute.

If a randomly selected adult is known to be a trained endurance athlete, what is the probability that their resting heart rate is below 60 beats per minute?

INDEPENDENCE

Recall: Two events A and B are independent if $P(AB) = P(A) \cdot P(B)$.

In the case they are not independent, then

$$P(AB) = P(A | B)P(B) \quad \text{General Multiplication Rule}$$

This follows from the fact that

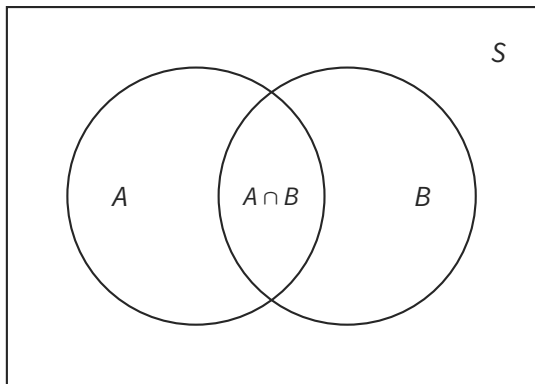
$$P(A | B) = \frac{P(AB)}{P(B)}$$

It can be shown, if A and B are independent then

$$P(A | B) = P(A)$$

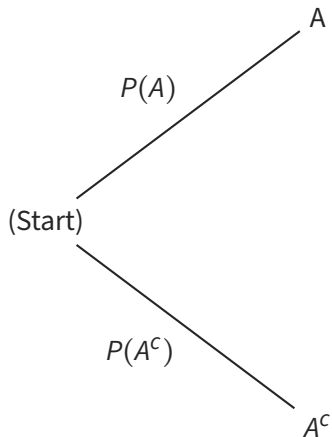
Knowing B has occurred, provides no additional information on A occurring.

PARTITIONING EVENTS

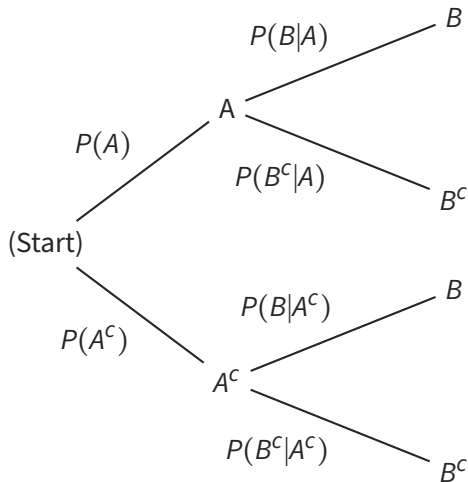


TREE DIAGRAMS

A **tree diagram** is a visual way to show the possible outcomes of a sequence of events and their probabilities



For conditional probability, each branch shows the probability of an outcome given what has already happened



GUIDED PRACTICE

A teacher observes that one of students is late to class 20% of the time, and when this happens they have their tie with them 70% of the time. When they are on time they have their tie 90% of the time.

Construct a tree diagram and calculate the probability that, on a randomly chosen day, they have no tie

BAYES THEOREM

Named after Reverend Thomas Bayes, this theorem provides a useful way to calculate and update the probability of an event based on new evidence

$$P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B | A)P(A)}{P(B)}$$

The numerator follows directly from the definition of conditional probability and noting that

$$P(A \cap B) = P(B | A)P(A)$$

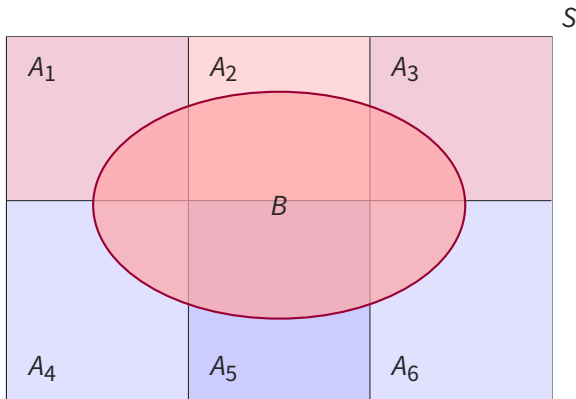


Reverend Thomas Bayes Magnet by STATISTICAT

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TOTAL LAW OF PROBABILITY

Consider the set of events $(A_1, A_2, \dots, A_n)$ partition the sample space, that is they are mutually exclusive and together cover all possible outcomes.



How can we compute the probability of B ?

Total Law of Probability: If a set of event $(A_1, A_2, \dots, A_n)$ partition the sample space (mutually exclusive and together cover all possible outcomes), the total probability of event B is given by

$$\begin{aligned} P(B) &= \sum_{i=1}^n P(B \cap A_i) \\ &= \sum_{i=1}^n P(B \mid A_i)P(A_i) \\ &= P(B \mid A_1)P(A_1) + \dots + P(B \mid A_n)P(A_n) \end{aligned}$$

BAYES THEOREM REVISITED

Bayes Theorem is based on obtaining $P(A | B)$ whenever $P(B | A)$ is known.

$$\begin{aligned} P(A | B) &= \frac{P(AB)}{P(B)} \\ &= \frac{P(B | A)P(A)}{P(B)} \quad \text{multiplication rule} \\ &= \frac{P(B | A)P(A)}{\sum_{i=1}^n P(B | A_i)P(A_i)} \quad \text{total law of probability} \end{aligned}$$

GUIDED PRACTICE

A teacher observes that a student is late to class 20% of the time. When the student is late, they have their tie with them 70% of the time. When they are on time, they have their tie 90% of the time.

Given that the student does not have their tie on a randomly chosen day, calculate the probability that the student was late.